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Dynamic response of multi-span arch bridge on spring supports subjected to moving vehicle

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A R T I C L E I N F O

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A B S T R A C T

To achieve the dynamic response of the arch bridge as the vertically curved bridge widely applied to the main structure of the bridge subjected to dynamic loads, the simulative methods based on engineering software have been used more popular than the analytical methods or finite element methods because of those complications. It is difficult to propose a general solution for analyzing dynamic response of the arch bridge due to different dynamic loads applied widely and easily in the design practice. Hence, this paper proposes the simplest model for the dynamic response of a multi-span arch bridge subjected to moving vehicle. The arch bridge modeled as a multi-span uniform arch beam resting on elastic spring supports is disjointed based on finite element method. The moving vehicle is described by two masses corresponding to car body and wheel. And then, the governing equation of motion of the bridge-vehicle interaction is derived based on dynamic balance principle and solved by Newmark method in the time domain. The accuracy of the algorithm is verified by comparing the numerical results with the other numerical results in the literature. Therefore, the influence of characteristic parameters of the multi-span arch bridge-vehicle interaction such as vertical curved of an arch bridge, the elastic stiffness of the support and the property parameters of the moving vehicle on the dynamic response of the system structure are investigated detail. The numerical results showed that those parameters affect significantly on the dynamic response of the multi-span arch bridge-vehicle interaction. It can be also seen that this study has meaningful practice in the problems of design and analysis response of the arch bridge due to moving traffic load.

1. Introduction

The problem models of bridge-vehicle dynamic interaction have been attracted the attention of many researchers in last few decades because of increasing the demand for using high-speed and heavy vehicles. In those problem models, there are many different approaches for analyzing dynamic response of the bridge-vehicle interaction. One of the most popular approaches for analyzing dynamic response of the bridge-vehicle interaction considers the bridge structures as a straight beam with uniform or non-uniform section due to moving loads or vehicles (Deng and Wang, 2015; Neves et al., 2012; Tanuja and Animesh, 2017; Yang and Yau, 1997; Yang et al., 2004). In those studies, the bridge beams were described as simple support or multi-span beam using Euler-Bernoulli or Timoshenko theory based on analytical solutions or finite element methods for analyzing dynamic response of the bridge-vehicle interaction.

Besides, the dynamic response of the curved bridge had quite many researchers in recent decades. The curved bridges were described by a curved deck or beam subjected to many different moving load types (Huang et al., 2000; Howson and Jemah, 1999; Kawakami et al., 1995; Lee et al., 2002; Reis and Pala, 2009; Montalvão e Silva and Urgueira, 1988; Wang et al., 1980; Wang and Sang, 1999; Yang et al., 2001; 2008; 2009). To analyze dynamic response of the curved bridge, both the analytical methods and the finite element methods were also

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employed. But, the analytical methods seem more popular than the finite element methods for analyzing dynamic response the curved bridge.

Additionally, the arch bridge as the vertically curved bridge has been widely applied to the main structure of the bridge due to different dynamic loads. There are many different types of arch bridge depend on the location of the bridge deck such as deck arch if the bridge deck lies above the arch, through arch if the deck lies in the spring line of the arch and half-through arch if the deck is elevated and placed between the spring line and the arch crown (Malm and Andersson, 2006; Wu and Chiang, 2004; Yue et al., 2005). To achieve the dynamic response of the arch bridge, the simulative methods based on engineering software have been used more popular than the analytical methods or finite element methods because of those complications. It can be seen that it is difficult to propose a general solution for analyzing dynamic response of the arch bridge due to different dynamic loads applied widely and easily in the design practice.

Hence, this study tries to propose the simplest model based on finite element method for the dynamic response of arch bridge subjected to moving vehicle. The arch bridge is described by multi-span arch beam based on Euler-Bernoulli theory resting on elastic spring supports. The system of the multi-span arch bridge and the moving vehicle as two degrees of freedom system is disjointed based on finite element method, and then the governing equation of motion of the bridge-vehicle interaction is derived based on dynamic balance principle and solved by Newmark method in the time domain in the formulation section. Therefore, the influence of characteristic parameters of the multi-span arch bridge-vehicle interaction such as vertical curved of an arch bridge, the elastic stiffness of the support and the property parameters of the moving vehicle on the dynamic response of the structural system are studied detail in the numerical investigation section.

2. Formulation

2.1. The multi-span arch bridge-vehicle model

The arch bridge subjected to moving the vehicle is modeled as a multi-span beam based on Euler-Bernoulli theory resting on elastic spring supports considered as rubber supports having rotational, vertical and horizontal linear elastic stiffness denoted by K_θ, K_h, and K_v, plotted in Fig. 1.

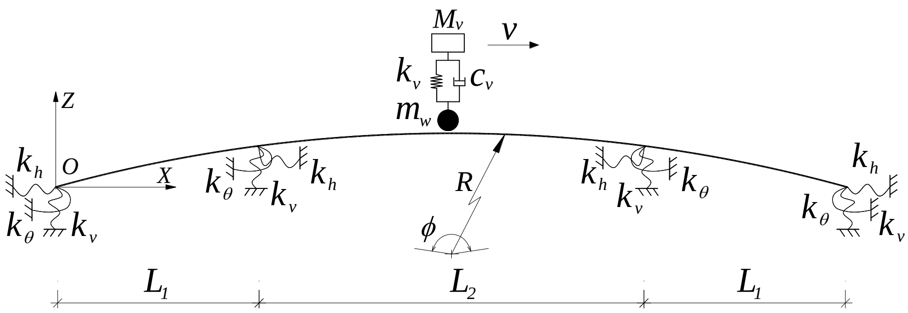


Fig. 1: The arch bridge-vehicle interaction model

The moving vehicle is regarded as a two-node system, with one node associated with each of two concentrated masses having the stiffness and damping coefficients of the moving vehicle denoted by k_v and c_v, and the mass of the wheel and the mass lumped from the car body by m_w and M_v, respectively (Neves et al., 2012). By assuming the no-jump condition for the moving vehicle, the equation of motion of the vehicle system can be written as follows:

$$\begin{bmatrix} M_v & 0 \\ 0 & m_w \end{bmatrix}\begin{Bmatrix} \ddot{z}_v \\ \ddot{z}_w \end{Bmatrix} + \begin{bmatrix} c_v & -c_v \\ -c_v & c_v \end{bmatrix}\begin{Bmatrix} \dot{z}_v \\ \dot{z}_w \end{Bmatrix} + \begin{bmatrix} k_v & -k_v \\ -k_v & k_v \end{bmatrix}\begin{Bmatrix} z_v \\ z_w \end{Bmatrix} = \begin{bmatrix} 0 \\ f_c - (M_v + m_w)g \end{bmatrix} \quad (1)$$

where f_c is the contact force, given by

$$f_c = (m_w + M_v)g + m_w\ddot{z}_w + M_v\ddot{z}_v \quad (2)$$

where z_v and z_w denote the vertical displacements of two nodes, respectively.

2.2. Finite element method

It is assumed that the divided element number is enough large to be able to consider each arch bridge element as a straight beam element, plotted in Fig. 2.

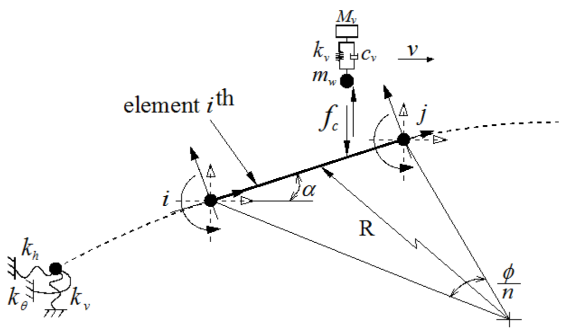


Fig. 2: The arch bridge element

Then, the geometric relationships of the parameters of arch bridge element are determined as follows

$$l = \frac{\emptyset}{n}R, \alpha = \arcsin\frac{\cos\left(\frac{\emptyset}{2}-i\frac{\emptyset}{n}\right)-\cos\left(\frac{\emptyset}{2}-(i-1)\frac{\emptyset}{n}\right)}{l} \tag{3}$$

where α is the angle between the axis of the arch bridge element and horizontal direction, R and $\emptyset$ denote curvature radius and angle of the arch bridge, l and n denote the length of the arch bridge element and divided element number of the multi-span arch bridge, respectively.

Based on the finite element method of the Euler-Bernoulli theory, the arch bridge is disjointed based on the finite element method of the two-node beam element. Each node has three global degrees of freedom including two displacements in global axes and one rotation. And then, the matrices of the arch bridge element in the global coordinates is obtained by a combination of the matrix of the bar element and the beam element, is given by

$$\mathrm{K}_e = \mathrm{L}^T\mathrm{K}'_e\mathrm{L}, \mathrm{M}_e = \mathrm{L}^T\mathrm{M}'_e\mathrm{L} \tag{4}$$

where L is the local-global transformation matrix of the arch bridge element, K'_e and M_e denote the stiffness matrix and the mass matrix of the arch bridge element in the local coordinates. Those matrices are presented in many previous works rated to the finite element method, are given by

$$\mathbf{K}'_e = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ & & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ & & & \frac{EA}{l} & 0 & 0 \\ & & & & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ syms & & & & & \frac{4EI}{l} \end{bmatrix} \tag{5}$$

$$\mathbf{M}'_e = \rho A l \begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ & \frac{13}{35} & \frac{11}{210}l & 0 & \frac{9}{70} & -\frac{13}{420}l \\ & & \frac{1}{105}l^2 & 0 & \frac{13}{420}l & -\frac{1}{140}l^2 \\ & & & \frac{1}{3} & 0 & 0 \\ & & & & \frac{13}{35} & -\frac{11}{210}l \\ syms & & & & & \frac{1}{105}l^2 \end{bmatrix} \tag{6}$$

$$\mathbf{L} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

2.3. The governing equation

Based on dynamic balance principle, the governing equation of the arch bridge-vehicle interaction element at each time step can be expressed as follows

$$\mathbf{M}_e\ddot{\mathbf{q}}_e + \mathbf{C}_e\dot{\mathbf{q}}_e + \mathbf{K}_e\mathbf{q}_e = \mathbf{F}_e \tag{8}$$

where $\mathbf{F}_e$ is the vector of consistent nodal forces caused by the contact force including both vertical and horizontal force in the local coordinates, given by

$$\mathbf{F}_e = \mathbf{F}_{e,v} + \mathbf{F}_{e,h} \tag{9}$$

where

$$\mathbf{F}_{e,v} = -\delta(\cdot)\mathbf{N}_v^T f_c\cos\alpha, \mathbf{F}_{e,h} = -\delta(.)\mathbf{N}_h^T f_c\sin\alpha \tag{10}$$

where $\delta(\cdot)$ is the Dirac-delta function, $\mathbf{N}_h$ and $\mathbf{N}_v$ denote the shape function of the bar element and the beam element given by

$$\begin{aligned} \mathbf{N}_h &= [N_1 \quad 0 \quad 0 \quad N_4 \quad 0 \quad 0] \\ \mathbf{N}_v &= [0 \quad N_2 \quad N_3 \quad 0 \quad N_5 \quad N_6] \end{aligned} \tag{11}$$

where N_i can be expressed as follows

$$\begin{aligned} &N_1 = 1-\frac{x}{l}, N_2 = 1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3}, N_3 = x-\frac{2x^2}{l}+\frac{x^3}{l^2} \\ &N_4 = \frac{x}{l}, N_5 = \frac{3x^2}{l^2}-\frac{2x^3}{l^3}, N_6 = -\frac{x^2}{l}+\frac{x^3}{l^2}. \end{aligned} \tag{12}$$

By assembling the element matrices in the global coordinates, the governing equation of motion of the multi-span arch bridge-vehicle interaction can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} \tag{13}$$

where $\mathbf{u}$ is the global displacement vector, $\mathbf{F}$ denotes the global load vector, $\mathbf{M}$ denotes the global mass matrix and $\mathbf{K}$ is the global stiffness matrix given by

$$\mathbf{K} = \mathbf{K}_b + \mathbf{K}_s \tag{14}$$

where $\mathbf{K}_b$ is the global stiffness matrix of the multi-span arch bridge assembled from the stiffness matrices of the arch bridge element and $\mathbf{K}_s$ denotes the stiffness matrix of the elastic linear spring supports corresponding with a degree of freedom of each elastic linear spring support in the global coordinates expressed as follows

$$\mathbf{K}_s = diag[K_h \quad K_v \quad K_\theta \quad \ldots \quad K_h \quad K_v \quad K_\theta] \tag{15}$$

and the damping matrix $\mathbf{C}$ can be obtained by adopting Rayleigh damping as

$$\mathbf{C} = \alpha_0\mathbf{M} + \alpha_1\mathbf{K} \tag{16}$$

where a_0 and a_1 denote Rayleigh damping coefficients (Chopra, 2016).

The above dynamic equation is used for studying the dynamic response of the bridge-vehicle interaction and is solved by means of the direct integration method based on Newmark algorithm, plotted in Fig. 3. And then, the computer program using MATLAB language is developed by the authors based on the above flowchart and the accuracy of the algorithm is verified by comparing the numerical results with the other ones in the literature.

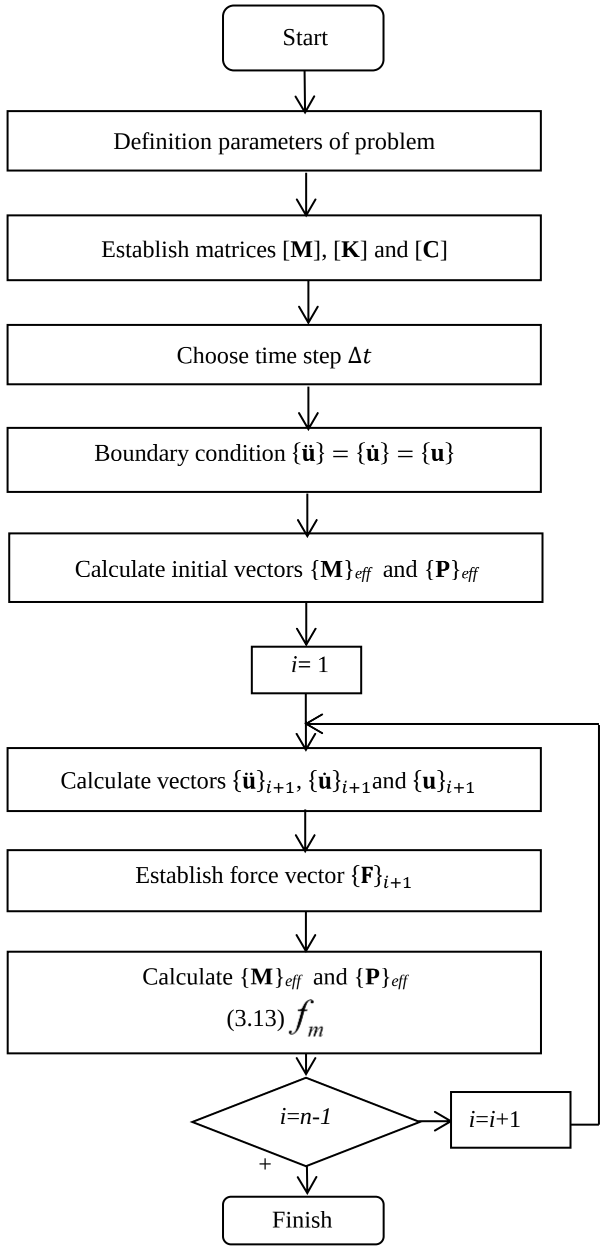


Fig. 3: Flowchart for analyzing the dynamic response of the arch bridge-vehicle interaction

3. Numerical results

3.1. Verification

In the first verification, a simple support arch bridge due to a concentrated force at the middle of the arch bridge is investigated. The static vertical displacement of the arch bridge is compared with the results obtained from simulation, shown in Fig. 4.

In the next verification, the time history of the vertical displacement at the middle of the straight simple support bridge subjected to the moving vehicle is compared with the results obtained by Neves et al. (2012), plotted in Fig. 5 and the time history of the displacements of the body car obtained in both this paper and Neves et al. (2012) is also presented in Fig. 6.

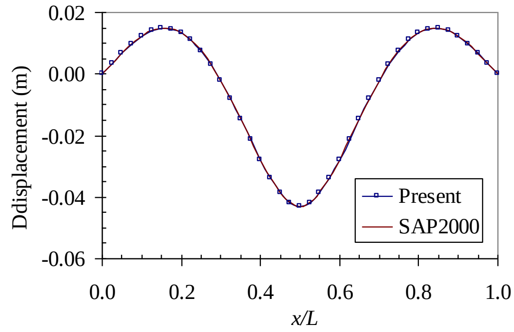


Fig. 4: The static vertical displacement of the simple support arch bridge

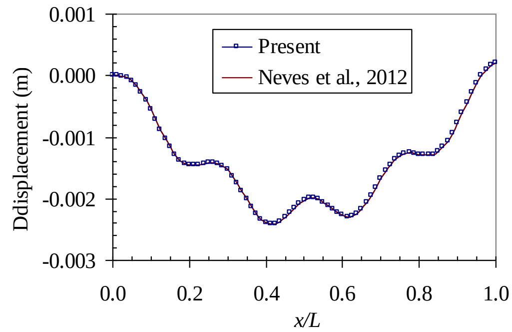


Fig. 5: The time history of vertical displacement at the middle of the straight bridge

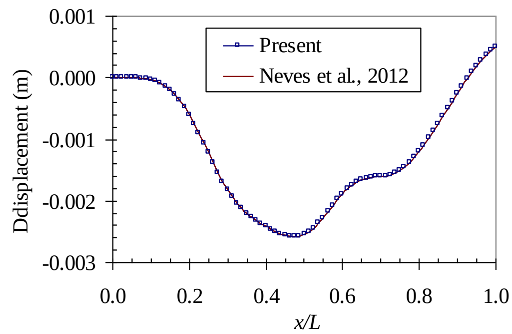


Fig. 6: The time history of displacement of body car

It can be seen that this numerical verification based on the suggested formulation is quite good agreement with numerical results in the literature. Therefore, the algorithm which used to analyze the influence of characteristic parameters of the multi-span arch bridge-vehicle interaction on the dynamic response of the structural system is reliable. The properties of the multi-span arch bridge-vehicle interaction are given in Table 1.

3.2. Effect of geometric parameters

In the first investigation, the influence of the geometric parameters of the multi-span arch bridge such as rotation angle θ and curvature radius R on the dynamic response of the arch bridge-vehicle interaction is studied for various the moving velocities. Figs. 7 and 8 present the time history of vertical displacement at each middle span in the linear elastic spring supports case with the moving velocity v=25 ms^{-1} and v=50 ms^{-1}, respectively. It can be seen that with an increase of the curvature angles decrease time history of vertical displacements of the left and right mid-span, shown in Fig. 7(a, c) and Fig. 8(a, c). But, the time history of the vertical displacements at the middle of the mid-span increase with an increase of the curvature angles, plotted in Fig 7(b) and Fig. 8(b). It is also commented that when the supports are not idealized, the sliding displacement of the supports is significant and decrease the general stiffness of the arch bridge. Hence, with an increase of curvature angles increase the time history of the vertical displacement of the mid-span.

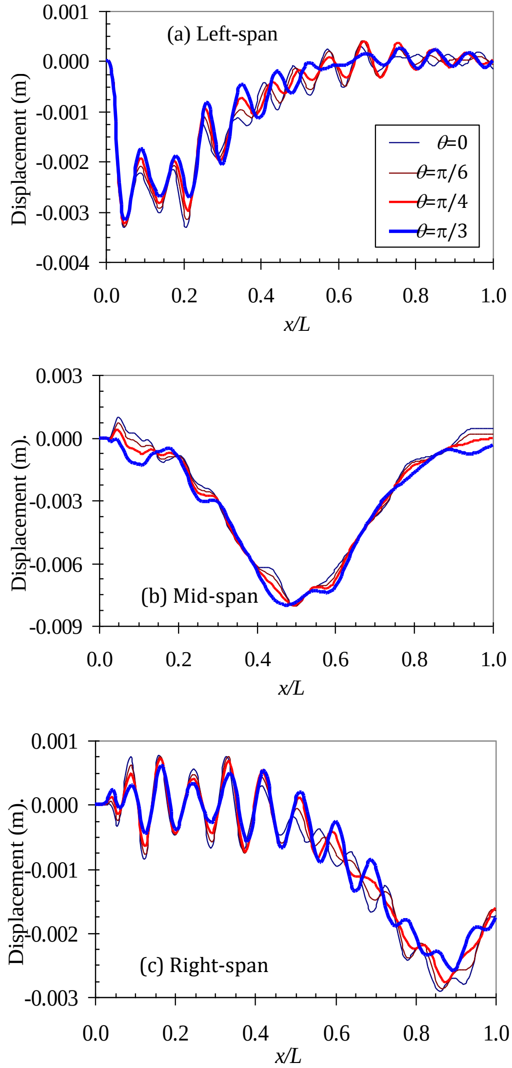
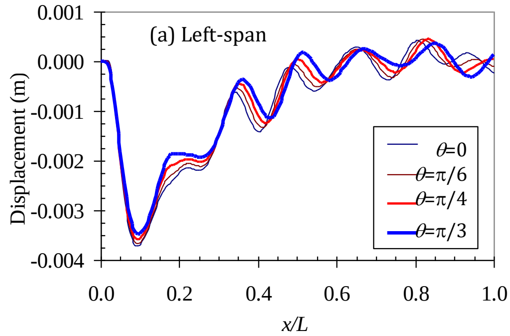


Fig. 7: The time history of vertical displacement at the middle span with the moving velocity v=25 ms-1



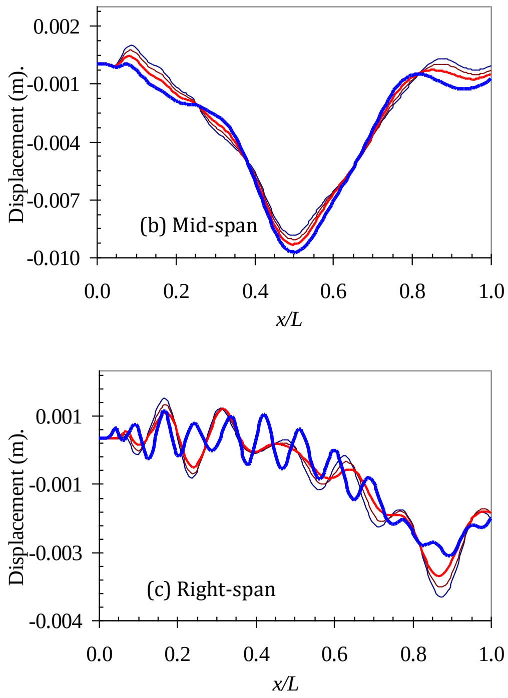


Fig. 8: The time history of vertical displacement at the middle span with the moving velocity v=50 ms-1

Table 1: The properties of bridge-vehicle interaction

Parameters	Unit	Value
The three-span arch bridge		
Curvature length L_1	m	20
Curvature length L_2	m	40
Young's modulus E	Nm^{-2}	2.87E9
Moment of inertia I	m^4	2.9
Mass per unit length ρ	kgm^{-1}	2303
Horizontal stiffness of support K_h	Nm^{-1}	8.1E5
Vertical stiffness of support K_v	Nm^{-1}	1.7E7
Rotation stiffness of support K_θ	Nm/rad	2.5E7
Damping ratio ξ		0.02
The moving vehicle		
Body mass M_v	kg	5750
Spring stiffness k_v	Nm^{-1}	1.595E6
Damping coefficient c_v	Nsm^{-1}	4.5E3
Wheel mass m_w	kg	250

Those things are completely different with the dynamic response of the multi-span arch bridge in the idealized supports, with an increase of the curvature angles decrease the time history of vertical displacements of the multi-span arch bridge, shown in Fig. 9. It can be seen that the response agrees completely with the characteristic behavior of the arch bridge in the idealized supports case. Hence, the stiffness support is truly important parameters for analyzing dynamic response of the arch bridge due to dynamic loads.

Additionally, the time history of the sliding displacement of the supports are discussed for various curvature angles, plotted in Figs. 10 and 11. It can be clearly seen that with an increase of the curvature angles increase the sliding displacement of the supports. When those angles increase cause increase curvature of the arch bridge and then it increases horizontal force in the spring supports as increase the sliding displacement of the supports. If the sliding displacements are enough large, the destruction of the part of the support bridge will increase with an increase of the curvature angles.

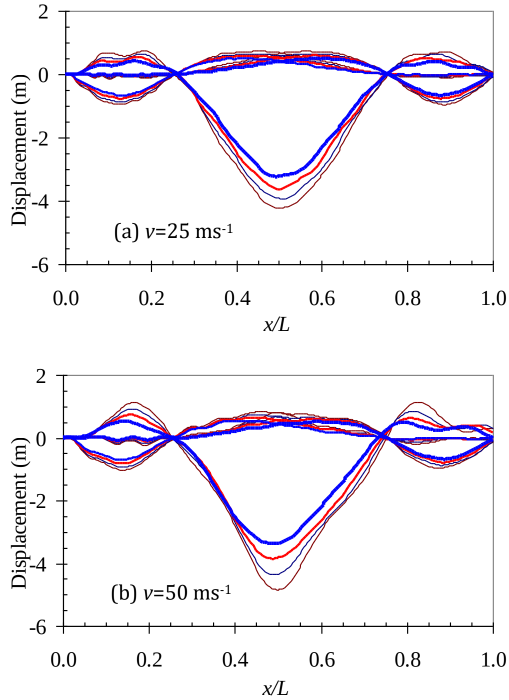
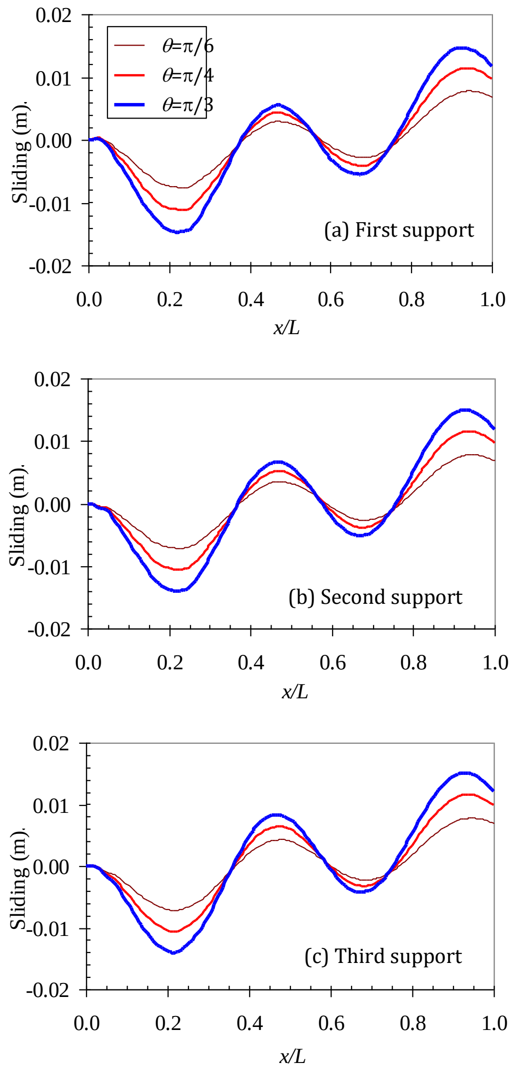


Fig. 9: The time history of vertical displacement at the middle spans in idealize supports case



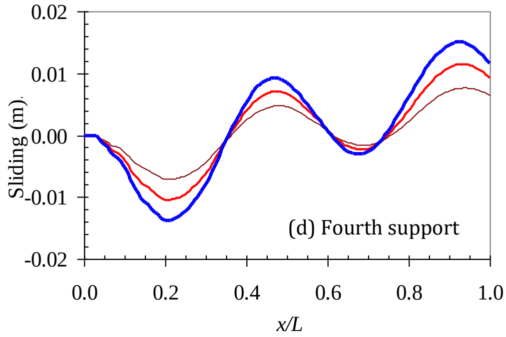


Fig. 10: The time history of sliding displacement at the spring supports with v=25 ms^{-1}

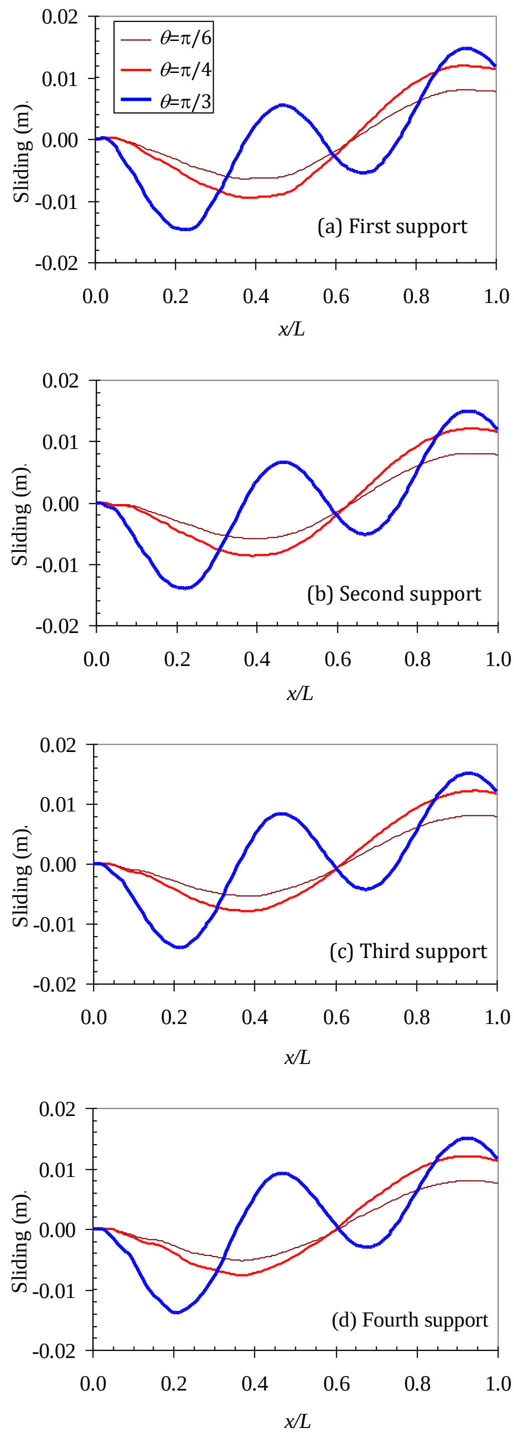


Fig. 11: The time history of sliding displacement at the spring supports with v=50 ms^{-1}

3.3. Effect of stiffness support parameters

The next investigation, the effects of the elastic spring support stiffness on the dynamic response of the arch bridge are also presented with the moving velocity v= 25 ms^{-1} and the various curvature angle $\theta=\pi/4$. Fig. 12 plots the influence of the vertical stiffness of the elastic spring supports on the time history of the vertical displacements at the middle of each mid-span.

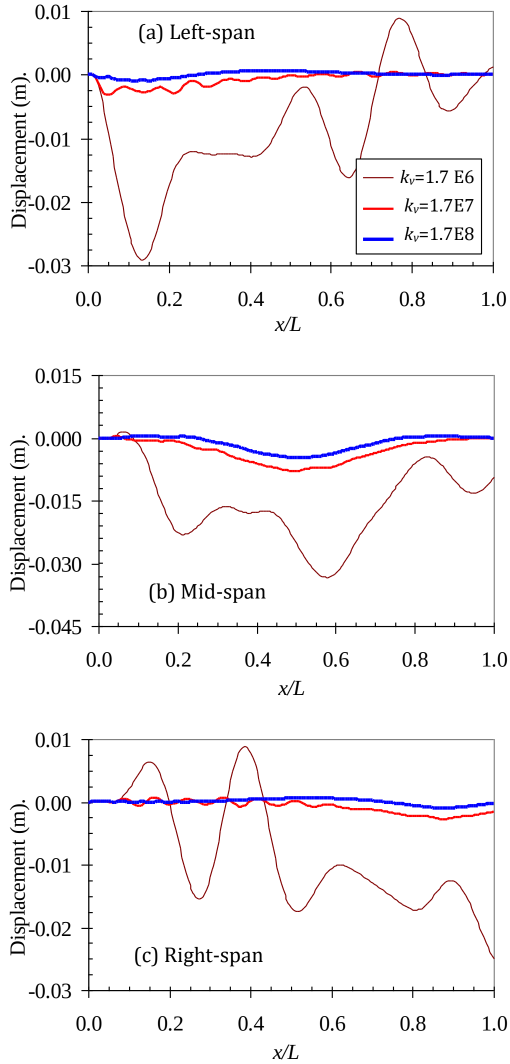


Fig. 12: The influence of vertical stiffness of spring support on vertical displacement at the middle span

It can be seen that the stiffness support effects significantly on the dynamic response of the arch bridge, with an increase of the vertical stiffness of support as increase the general stiffness of the arch bridge decreases the time history of the vertical displacement of each span (Fig. 12).

Besides, the sliding displacement of the supports also depends significantly on the horizontal stiffness of the spring supports, shown in Fig. 13. When the horizontal stiffness of the supports increase enough large, the spring supports can be considered as idealized supports and then the sliding displacements of the support decrease clearly with an increase of those. Hence, it can be seen that the stiffness support including vertical and horizontal stiffness effects directly on the dynamic response of the multi-span arch bridge, those parameters need to be able to consider carefully for analyzing dynamic response of the arch bridge due to dynamic loads.

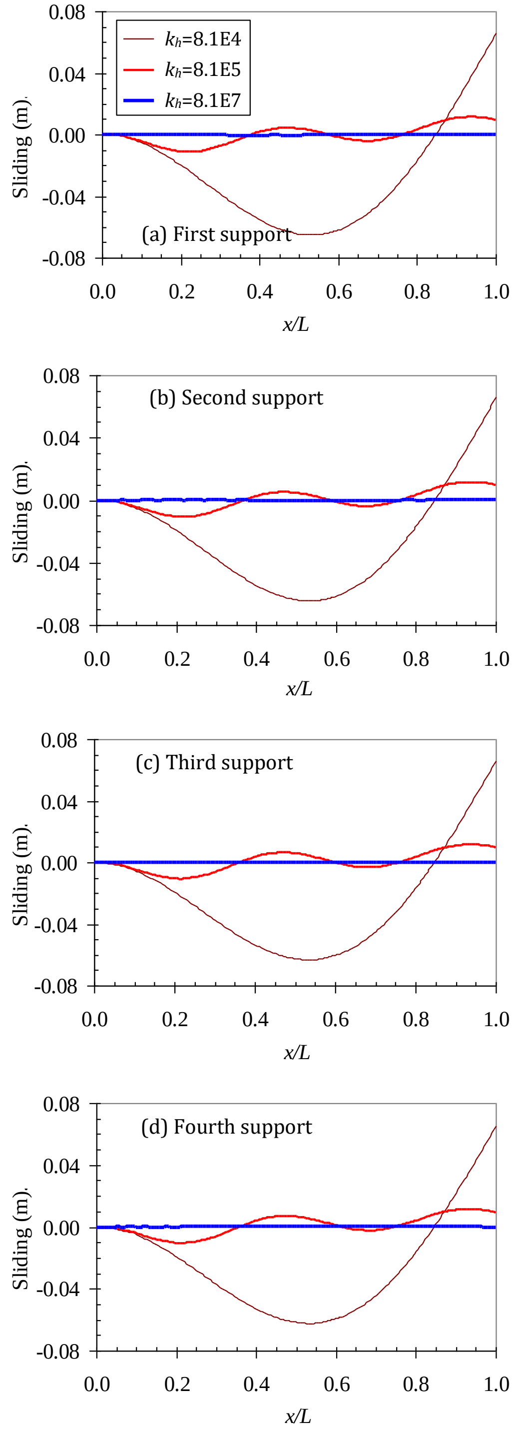


Fig. 13: The influence of horizontal stiffness of spring support on sliding displacement

3.4. Effect of moving velocity

It can be seen that the geometric and stiffness parameters of the multi-span arch bridge almost effect significantly on the dynamic response of the arch bridge-vehicle interaction. At the same time, another parameter having quite an important role for analyzing dynamic response of the arch bridge-vehicle interaction is the moving vehicle velocity. The influence of this parameter will be investigated in the next section, plotted from Figs. 14 to 20. The numerical investigations show that the moving velocities affect significantly on the dynamic response of the arch bridge for various parameters of the arch bridge-vehicle interaction. It can be also seen that with an increase of the curvature angles increase the vertical displacement of the middle of the mid-span in the spring supports case, shown in Fig. 14(b).

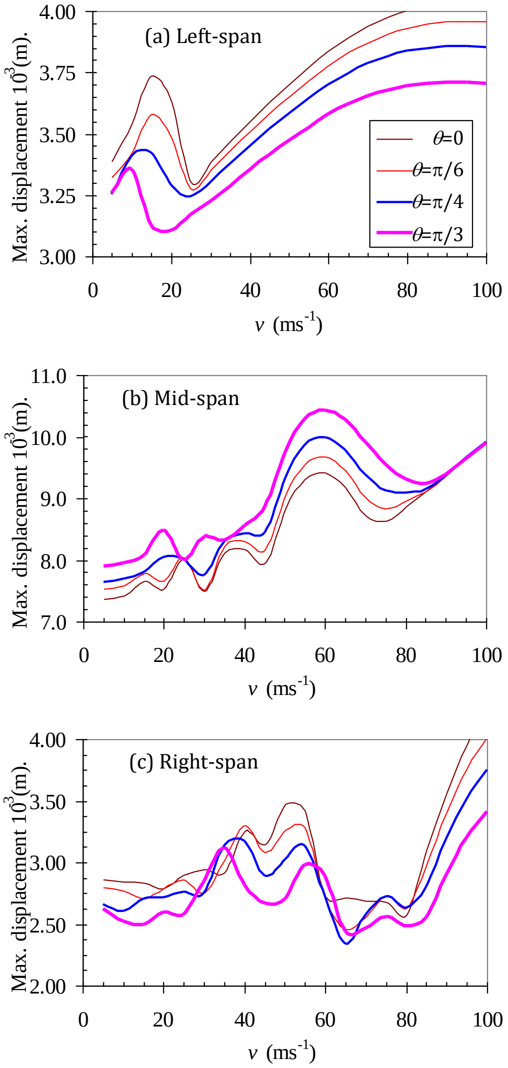
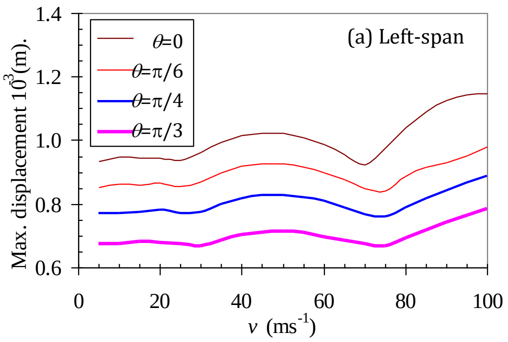


Fig. 14: The influence of moving velocity on maximum vertical displacement in the spring supports case



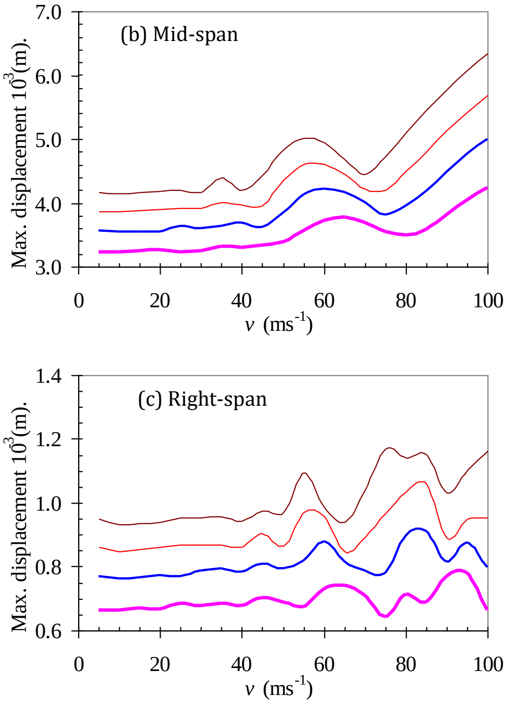
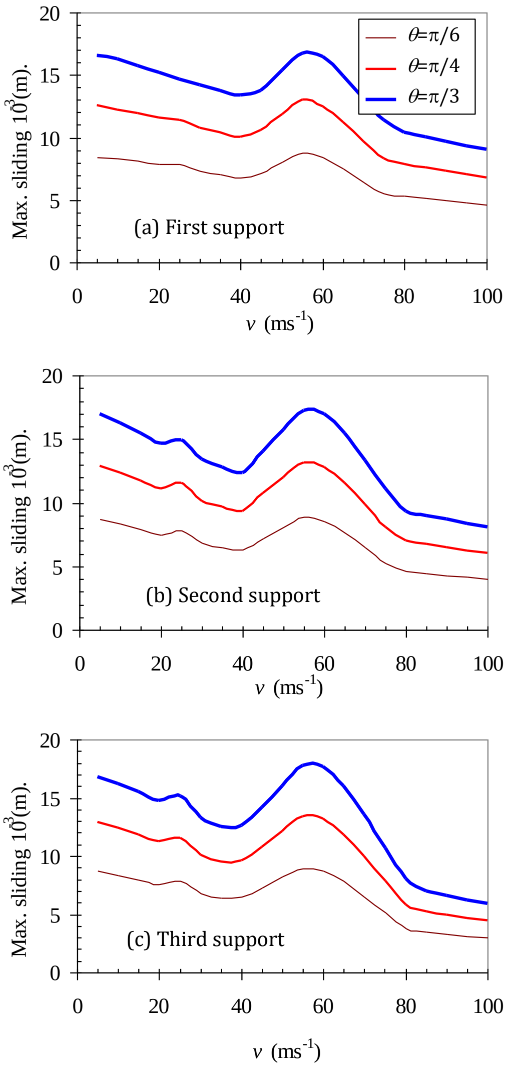


Fig. 15: The influence of moving velocity on maximum vertical displacement in the idealized supports case



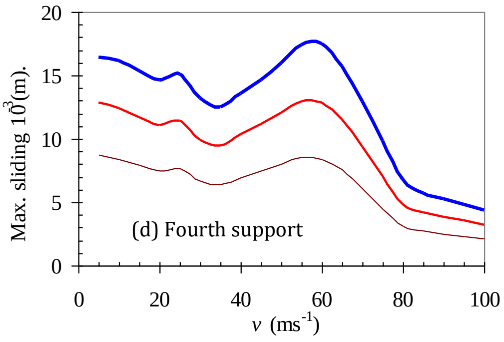


Fig. 16: The influence of moving velocity on maximum sliding at the spring supports

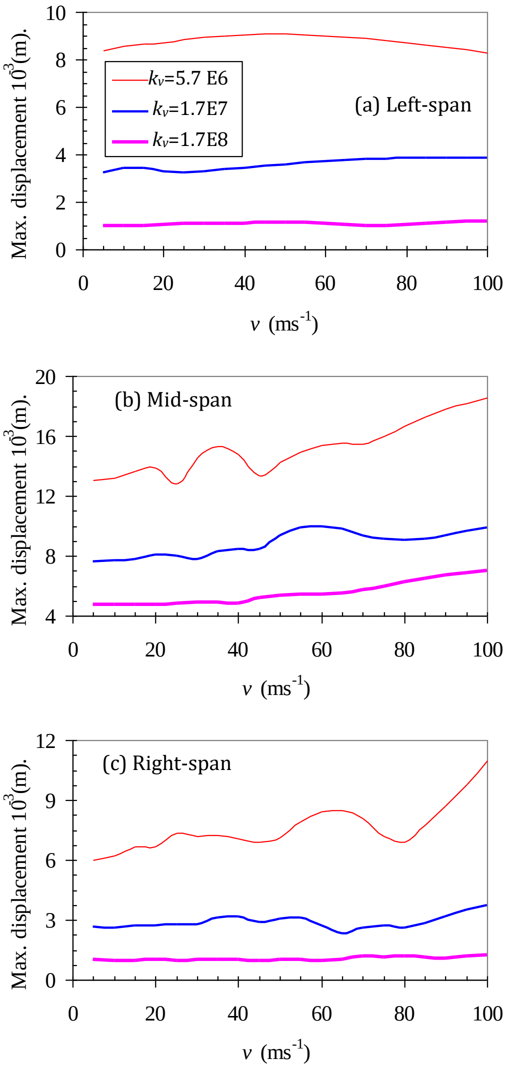


Fig. 17: The influence of moving velocity on maximum vertical displacement at the middle span

But, the influence of those parameters is opposite in the idealized supports case, this agrees completely with the behavior of idealized multi-span arch bridge due to dynamic loads, plotted in Fig. 15. Besides, with an increase of curvature angles also increase the sliding displacement of the support for various velocities, in Fig. 16.

Besides, the investigation results also show that the stiffness supports such as vertical and horizontal stiffness effect directly on the general stiffness of the multi-span arch bridge, with an increase of the stiffness supports will cause increase of the general stiffness of the arch bridge and then the dynamic response of the bridge-vehicle interaction also decreases with an increase of those parameters, are plotted in Figs. 17 and 18.

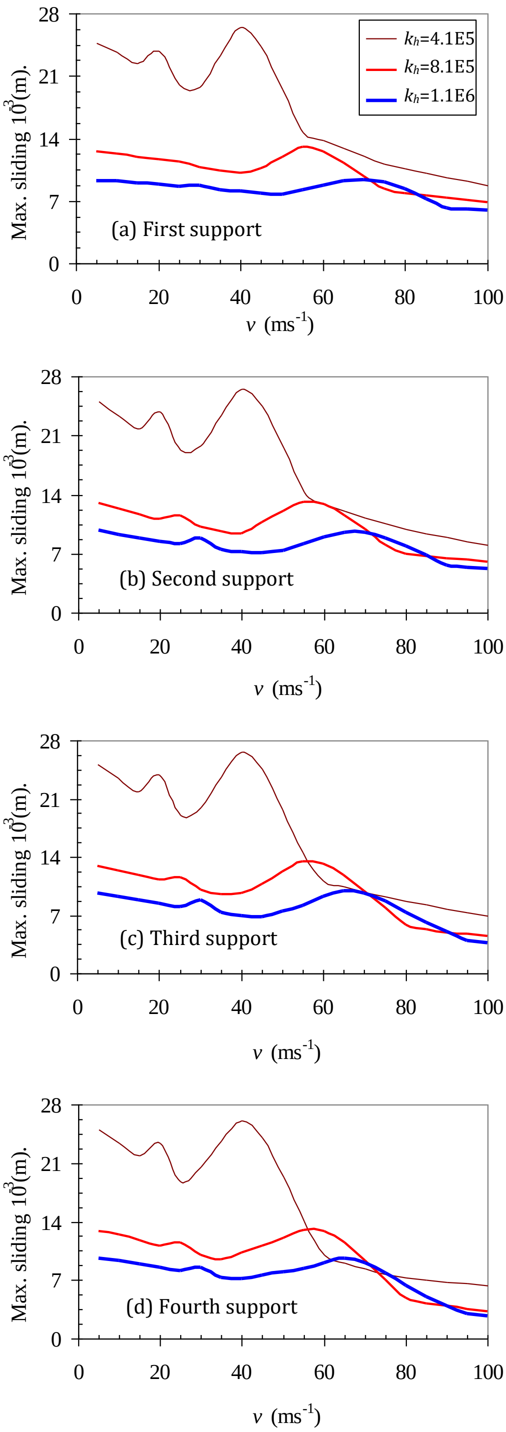


Fig. 18: The influence of moving velocity on maximum sliding at the spring supports

Additionally, the influence of rotation stiffness of the spring support on the dynamic response of the arch bridge is studied in Figs. 19 and 20. When the rotation stiffness of the support increases, it also increases the general stiffness of the multi-span arch bridge. Especially, the rotation at the bridge supports will increase and then it decreases the dynamic response of the arch bridge with an increase of rotation stiffness, shown in Fig. 19.

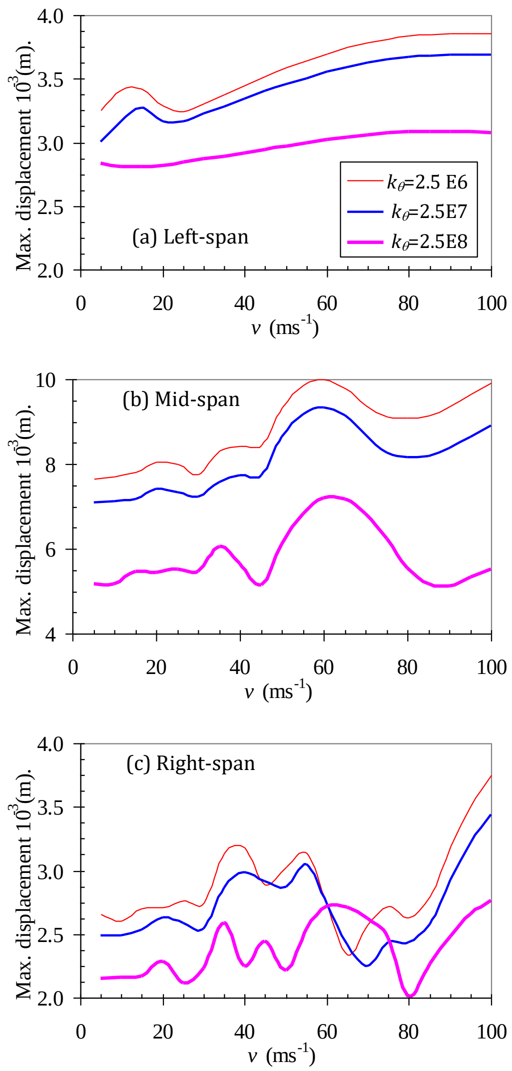


Fig. 19: The influence of moving velocity on maximum vertical displacement at the middle span

But, the sliding displacements of the spring supports almost do not depend on the rotation stiffness, it does not affect significantly on the sliding response of the supports for several of the moving velocities, plotted in Fig. 20.

It can be seen that the property parameters of the multi-span arch bridge effect significantly on the dynamic response of the arch bridge-vehicle interaction. With each parameter has a different influence on the sliding response of the supports and dynamic displacements of the arch bridge. Hence, the chosen parameters have an important role for analyzing dynamic response of the arch bridge-vehicle interaction.

4. Conclusion

Based on the results for the dynamic response of the multi-span arch bridge subjected to moving vehicle, some conclusion is drawn as follows:

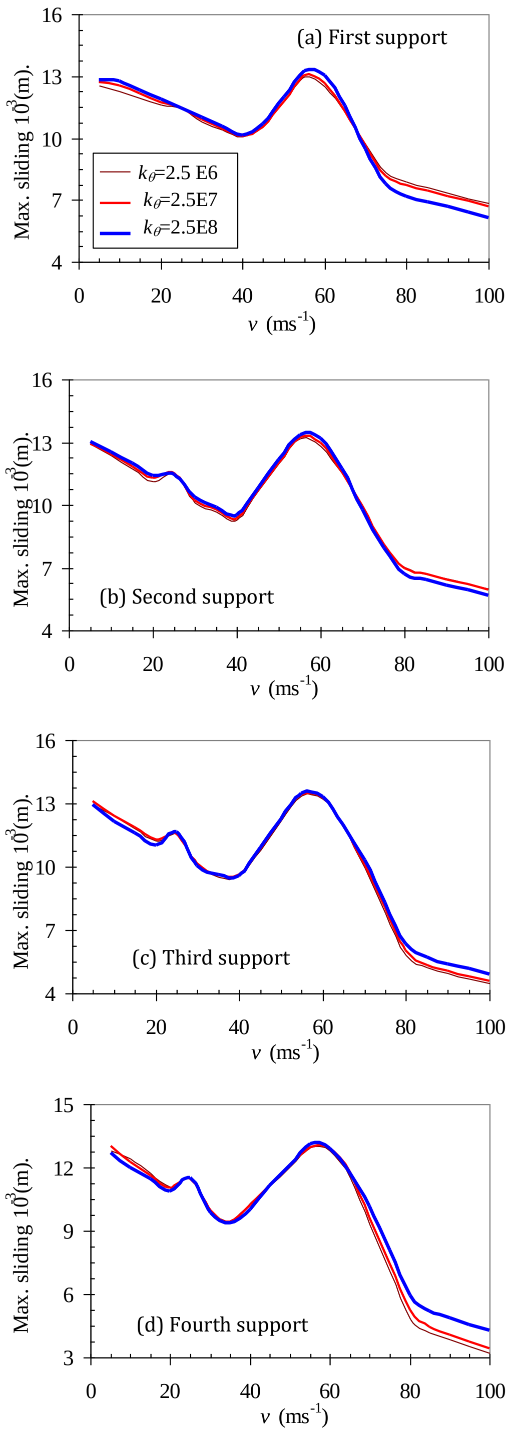


Fig. 20: The influence of moving velocity on maximum sliding displacement at the spring supports

- The formulation of finite element method for analyzing dynamic response of the multi-span arch bridge on the spring supports subjected to moving the vehicle is presented detail in this paper.
- The influence of the geometric parameters on the vertical dynamic displacements and the sliding of the spring supports are significant. With an increase of those increase, the sliding displacement of the spring supports and the vertical displacement of the arch bridge, but this is opposite in the idealized supports case.

- The stiffness parameters of the spring supports have an important role in the problem model of analyzing dynamic response of the arch bridge. The dynamic response of the arch bridge such as the sliding and vertical displacement decreases with an increase of the horizontal and vertical stiffness of the spring supports for various the moving velocities, respectively.

It can be seen that the multi-span arch bridge model on the spring support as true bridge supports subjected to moving vehicle quite agrees with the real bridge model due to traffic loads. Hence, this study can be considered as meaningful practice document for analyzing dynamic response of the arch bridge-vehicle interaction.

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